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Robotics in order picking: Evaluating warehouse layouts for pick, place, and transport vehicle routing systems

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Motivated by recent technological advances in mobile robotics, this paper explores a novel approach for warehouse order picking. In particular, this work considers two types of commercially available mobile robots – one that can grasp items from a shelf (a picker) and another (a transporter) that can quickly deliver all items from the pick list to the packing station. A new vehicle routing problem is defined which seeks to minimize the time to deliver all items from a pick list to the packing station, a problem termed the *pick, place, and transport vehicle routing problem.* A mixed integer linear programming formulation is developed to answer three related research questions. First, what combination of picker and transport robots is required to obtain performance exceeding traditional human-based picking operations? Second, how should the composition of the robot fleet be altered to affect the greatest performance improvements? Finally, what are the impacts of warehouse layout designs when coordinated mobile robots are deployed? An extensive numerical analysis reveals that, (1) increasing the number of cross aisles decreases system performance; (2) centrally located packing stations improve system performance; and (3) the average distance from each pick location to the packing station and the average distance between pick locations are effective metrics for identifying specific fleet modifications that are likely to yield system improvements.

Keywords: automated guided vehicle; facility layout; facility planning; robot applications; vehicle routing; warehouse design; warehousing systems

1. Introduction

It is estimated that order-picking operations can account for roughly 65% of the total operating cost, and 60% of all labor activities, in a warehouse (Ho, Su, and Shi 2008). Recent technological advances in mobile robotics promise to reduce these costs. For example, the recently-unveiled "Fetch" and "Freight" robots from Fetch Robotics, Inc., pictured in Figure 1 and detailed in Wise et al. (2016), have been marketed to the warehousing industry to improve order picking operations. Both robots are mobile and feature onboard laser scanners to detect and avoid obstacles. The Fetch robot is equipped with a camera system to identify the items to be picked and a gripper attachment for retrieving items from a storage rack. The smaller Freight robot is designed to transport items placed within a removable tote to a packing station where the items are prepared for shipping. Freight may be used in conjunction with Fetch, with Fetch placing items into Freight for transport in a *pick-and-place* process. Alternatively, Freight may be programmed to follow a human picker in a *follow-pick* system, allowing the human picker to stay within the warehouse while the faster robot transporter moves items to the packing station (in constrast to a traditional human-only system in which the human must also transport the picked items).

This paper discusses the potential timesavings that new robotics technologies may offer in order-picking operations. In particular, we consider the problem of collecting a pick list of items. Each item on the list occupies a space in the warehouse (a picking location), defined by specific two-

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(a) Fetch and Freight. (b) Fetch's robot hand for order retrieval

Figure 1.: Fetch and Freight (source: fetchrobotics.com).

dimensional coordinates and a height on the storage rack. A "picker" robot (e.g., Fetch) retrieves one item at a time, placing each item into a "transport" robot (e.g., Freight). This handoff is performed at the location where the item was retrieved. The transport robots deliver the items to a single packing station located within the warehouse. The objective is to minimize the time required to transport all items on the pick list from the warehouse to the packing station. We term this problem the *pick, place, and transport vehicle routing problem* (PPT-VRP). While this problem was inspired by Fetch and Freight, it is not specific to these particular robots; Locus Robitics (2018) and 6 River Systems (2018) sell similar robots for automated warehouses.

The availability of mobile picker and transport robots prompts a number of interesting research questions in the context of this order-picking problem. For example, what combination of picker and transport robots is required to obtain performance exceeding human-based picking operations, where human workers pick items and return all items described in the pick list to the packing station manually? Furthermore, how does this answer change if the robots have a constrained payload capacity (i.e., less than the size of the pick list)? To help answer these questions, a mixed integer linear programming (MILP) formulation of the PPT-VRP is proposed. Solutions to this problem describe the sequence of items to be collected by picker robots and establish the timing coordination between picker and transport robots.

This research also explores the benefits of a hybrid system in which humans are tasked to retrieve (pick) items while mobile robots are employed to transport these items to the packing station. Such a "follow-pick" system acknowledges that humans are (at least presently) more adept at identifying and grasping items from a storage shelf. It also leverages the faster travel speeds for the robotic transport unit. The proposed PPT-VRP formulation may be employed to determine the optimal number of transport robots to pair with a given number of human pickers.

Finally, this research examines the relationships between warehouse design and mobile robot picking operations. The impacts of the number of cross- and picking-aisles, as well as the location of packing stations within the warehouse, are explored. This paper also investigates the relative impacts of altering the mix and functionality of a fleet of pick-and-transport robots. This analysis provides insight into whether it is more beneficial to add an extra picker, add another transporter, increase the carrying capacity of a transporter, or increase the retrieval speed of a picker.

The remainder of this paper is organized as follows. Related literature is discussed in Section 2, followed in Section 3 by a formal mathematical programming model of the problem. This model is extended to consider combinations of robots and human pickers, as well as an environment in which only human pickers are available. We demonstrate empirically, via an extensive numerical analysis in Section 4, the impacts of various warehouse layouts and highlight the relative benefits associated with modifying the robot fleet's composition and capabilities. Finally, a summary and an overview of future research opportunities are provided in Section 5.

2. Related literature

There is a vast body of warehouse operation and layout design research, reviews of which may be found in de Koster, Le-Duc, and Roodbergen (2007), and Gu, Goetschalckx, and McGinnis (2007, 2010). One category of layout research involves unit-load warehousing, where large unit sizes (e.g., pallets) limit each picker to transporting one item at a time. Pohl, Meller, and Gue (2009) investigated optimization of warehouse layout structures with perpendicular aisles under a dual-command operation, where workers re-stock one item and retrieve another item in a route. Gue and Meller (2009) introduced novel non-traditional warehouse designs with diagonal cross aisles and non-parallel picking aisles. Ozturkoglu, Gue, and Meller (2012) proved that the non-traditional single cross aisle Chevron design outperforms others with more cross aisles. Other considerations of non-traditional layouts include turnover-based storage policies (Pohl, Meller, and Gue 2011), and multiple depots (Gue, Ivanovia, and Meller 2012; Ozturkoglu, Gue, and Meller 2014). These works seek the minimization of the expected traveling distance (or time) by changing warehouse layouts or storage policies. Order batching and detailed picker routing were not considered, as the focus was on unit-loads.

Another category of layout research involves batch picking, in which each picker may retrieve several items in a route. Roodbergen and Vis (2006) investigated the impact of warehouse layout parameters and routing policies on the expected traveling distances of picking tours. Parikh and Meller (2010) developed a throughput model that incorporates the vertical travel dimension for warehouses with varying lengths and heights of storage aisles. Thomas and Meller (2015) discussed the effects of various layout aspects upon labor hours. Roodbergen, Vis, and Taylor (2015) provided case studies for the determination of the layout parameters that reduce the average travel distance for order picking. These layout parameters included storage unit assignments, the number of cross aisles, warehouse shape, and aisle lengths. Similarly, a simulation-based statistical analysis of certain warehouse layout parameters was conducted by Shqair, Altarazi, and Al-Shihabi (2014).

As stated in Gu, Goetschalckx, and McGinnis (2007), proper order batching can also improve the efficiency of order retrieval. Warehouse order batching research focuses on splitting a set of orders (a pick list) into batches to ensure that all batches can be retrieved within a time window. Thus, the batch size is determined based on the required completion time for each batch (Petersen 2000; de Koster, Le-Duc, and Roodbergen 2007). Assuming given routing, heuristics for order batching problems have been studied under deterministic (c.f., Chen and Wu (2005); Gademann and Velde (2001, 2005); Hsu, Chen, and Chen (2005); Henn (2012); Pan, Shih, and Wu (2015)) and stochastic demands (c.f., Chew and Tang (1999); Le-Duc and de Koster (2007); Nieuwenhuyse and de Koster (2009); Henn (2012)). While these research works have provided efficient methodologies for order batching, they require known routes and consider given layout configurations.

Routing methodologies for order picking also play an important role in improving warehouse efficiency. The first model for optimal picker routing was proposed by Ratliff and Rosenthal (1983), which employed a traveling salesman problem (TSP) to minimize order retrieval time. Improved routing models were proposed by Scholz et al. (2016). Due to the computational complexity of this problem, a number of routing heuristics for order picking have been studied for problems of practical size (c.f., de Koster and Poort (1998); Chew and Tang (1999); Roodbergen and de Koster (2001b,a); Hwang, Oh, and Lee (2004); Theys et al. (2010)).

AGV-based warehousing systems, such as implementations of autonomous vehicle based storage and retrieval system (AVS/RS) for high-density storage warehouses, have received increasing interest. For example, Ferrara, Gebennini, and Grassi (2014) studied warehouses with pallet shuttles and laser guided vehicles which coordinate with each other for item passing at the intersections of aisles. Queueing models were applied to estimate the order retrieval time when adjusting batch sizes of fleets. Other queueing analytic studies have addressed dwell-points (c.f., Kuo, Krishnamurthy, and Malmborg (2007); Roy et al. (2012, 2015)), aisle locations (c.f., Roy et al. (2012, 2015)), and batch sizes of fleets (c.f., Fukunari and Malmborg (2009)). Simulation models were presented by Ekren et al. (2010) to analyze the effects of system operating policies and depot locations for AVS/RS. Saidi-Mehrabad et al. (2015) proposed a congestion free vehicle routing problem associated with job shop scheduling problems for AGVs transporting items from a warehouse to the manufacturing system via grid-based paths. Unlike human-based warehousing or the proposed PPT-VRP, AGV-based warehousing research assumes autonomous vehicles traveling along only designated rail guide-paths, which results in either warehouse zoning strategies or constrained routing policies.

In contrast to the Fetch & Freight robots, which retrieve individual items from the warehouse, Amazon's Kiva System robots transport entire racks to the packing area (e.g., Tam (2014); Wohlsen (2014)). Recent research on such "parts-to-picker" robotic environments includes Boysen, Briskorn, and Emde (2017a), Boysen, Briskorn, and Emde (2017b), Lamballais, Roy, and Koster (2017), and Bozer and Aldarondo (2018).

In a more general context, variants of the vehicle routing problem (VRP) are also closely related to the problem at hand. Of particular relevance is the VRP with multiple synchronization constraints (VRPMS), a review of which is provided by Drexl (2012) and efficient branch-and-cut algorithms are proposed by Drexl (2014). The VRPMS considers heterogeneous vehicles that must be coordinated to perform tasks such as load transfers or moving of truck trailers. The PPT-VRP extends the VRPMS to include queueing of delivery activities and vehicle recharging.

This paper aims to contribute to the literature in two key areas. First, the PPT-VRP represents a novel optimization problem for the coordinated routing of two types of heterogeneous vehicles (e.g., Fetches and Freights). This differs from existing routing methodologies for warehouse order retrieval, which consider routing for only individual pickers. It also removes restrictions found in AVS/RS problems in which vehicles are constrained to rail guide-paths or to particular zones. Second, this paper explores layout design guidelines for warehouses employing picker robots or combinations of human pickers and robot transporters. The impacts of warehouse layout parameters (e.g., the number of cross aisles, the number of picking aisles, and depot locations) are examined for different vehicle parameters (e.g., robot quantities, speeds, and capacities).

3. Problem definition and formulations

The PPT-VRP may be defined as follows. A pick list (collection) of items, denoted by the set $I = \{1, \ldots, |I|\}$, must be retrieved from the warehouse and delivered to a packing station (depot). The pick list, which has been pre-defined, may contain items from multiple customer orders. The objective of the PPT-VRP is to determine routes for all available picker and transporter vehicles such that the latest time at which all items from the pick list I are dropped off at the packing station; that is, to minimize the makespan.

Two types of specialized vehicles (mobile robots) are available; P represents the set of "picker" robots and D represents the set of "delivery" (or "transport") robots. The entire fleet of vehicles is thus given by the set $V = P \cup D$.

Vehicle "blocking" in aisles is not considered in this study. This is partially for the sake of enabling a tractable mathematical model. Although aisle congestion has been considered in the warehousing literature, it is typically under relatively restrictive conditions, such as S-shape routes in warehouses with uni-directional aisles (c.f., Gue, Meller, and Skufca (2006); Mowrey and Parikh (2014)), vehicles with designated or closed-loop paths (c.f., Ferrara, Gebennini, and Grassi (2014); Hong (2014); Hong, Johnson, and Peters (2012); Roy et al. (2015)). Furthermore, the aisle congestion research considers single picker types (c.f., Gue, Meller, and Skufca (2006); Hong, Johnson, and Peters (2012); Mowrey and Parikh (2014); Saidi-Mehrabad et al. (2015)), often with limited numbers (c.f., Chen et al. (2013); Hong (2014)). By contrast, this study considers a heterogeneous fleet of picker and transport robots that are free to travel along any paths. This flexibility leads to significantly more complicated vehicle routing formulations, even without provisions for conflict-free routing. However, from a practical perspective, the robots motivating this research are of a scale such that the typical aisle width can accommodate three of these robots in parallel. More details are provided in the numerical study in Section 4.1.

Each robot $v \in V$ may have a unique payload capacity, given by $\hat{w}_v$. The initial payload carried by a given robot is denoted by w'_v, while the weight of item $i \in I$ is given by $\bar{w}_i$. Each battery-powered vehicle $v \in V$ has an initial charge of $0 \leq c'_v \leq 1$, which represents the remaining percentage of battery life. Batteries are discharged at the rate of $0 \leq d_v \leq 1$ percent per unit time, which is assumed to be independent of payload. When a vehicle visits the depot, charging stations will re-charge batteries at a rate of $0 \leq r_v \leq 1$ percent per unit time. It is assumed that a sufficient number of charging stations are available, such that vehicles do not wait for charging access. Travel time from the packing station to the chargers is assumed to be negligible.

Three types of service time are separately identified. First, the time required for picker robot $v \in P$ to grasp item $i \in I$ from its location on the stocking shelf is denoted by $s^{\text{pick}}_{v,i}$. This accounts for differences in robot capabilities as well as additional time required to grasp items located far (vertically) from the robot's default pose. Similarly, let $s^{\text{place}}_{v,i}$ represent the time required for picker robot $v \in P$ to place item $i \in I$ into a delivery robot. Finally, the time required for all items transported by delivery robot $v \in D$ to be offloaded from the robot at the depot is given by s^{drop}_{v}. We assume that this time is independent of the number of items held by the robot, as the person collecting these items at the depot is expected to simply replace the used tote with an empty one. As delivery vehicles arrive at the depot they form a queue while waiting for their totes to be replaced. Table 1 summarizes the parameter notations of the PPT-VRP.

Table 1.: Summary of parameter notation.

I	Set of items comprising the pick list; $I = \{1, \ldots, \lvert I \rvert\}$.
P	Set of pickers.
D	Set of transporters; $P \cap D = \varnothing$.
V	Set of all vehicles in the fleet; $V = P \cup D$.
$\Delta^0_v = 0$	Initial location of vehicle $v \in V$.
Δ^*_v	Set of depot replicas for vehicle $v \in V$, where $\Delta^*_{v_1} \cap \Delta^*_{v_2} = \varnothing$ for all $v_1 \neq v_2$.
Δ^+_v	Set of nodes to which vehicle $v \in V$ may travel, such that $\Delta^+_v \subseteq \{I \cup \Delta^*_v\}$.
$\Delta^-_{v,j}$	Set of nodes that could be visited right after visiting node j, where $\Delta^-_{v,j\in I} \subseteq \{\Delta^0_v \cup I \backslash j \cup \Delta^*_v\}$, and $\Delta^-_{v,j\in\Delta^*_v} \subseteq \{\Delta^0_v \cup I \cup \max\{\Delta^*_v < j\}\}\ \forall\ v \in V$.
N	Set of nodes in the network; $N = 0 \cup I \cup_{v\in V} \Delta^*_v$.
$0 \leq c'_v \leq 1$	Remaining percentage of battery life for vehicle $v \in V$.
$0 \leq d_v \leq 1$	Discharging rate for vehicle $v \in V$.
$0 \leq r_v \leq 1$	Re-charging rate for vehicle $v \in V$.
F_v	Speed of vehicle $v \in V$.
$s^{\text{pick}}_{v,i}$	Time required for picker $v \in P$ to grasp item $i \in I$ from its storage location.
$s^{\text{place}}_{v,i}$	Time required for picker $v \in P$ to place item $i \in I$ into a transporter.
s^{drop}_{v}	Time required for transporter $v \in D$ to drop off a tote of items at the depot.
$\tau_{v,i,j}$	Time required for vehicle $v \in V$ to travel to node $j \in \Delta^+_v$ from node $i \in \Delta^-_{v,j}$.
w'_v	Initial payload capacity of vehicle $v \in V$.
$\hat{w}_v$	Overall payload capacity of vehicle $v \in V$.
$\bar{w}_i$	Weight of item $i \in I$.

3.1 *Representing the network structure*

An underlying network structure facilitates the characterization of vehicle movement. This network includes three types of nodes that represent (1) the initial location of each vehicle, (2) the locations of items to be retrieved from the warehouse, and (3) the location of the depot. We let $\Delta_v^0 = 0$ represent the initial location of vehicle $v \in V$. Although this node's label equals zero for all vehicles, it is not a requirement that each vehicle actually begin service at the same physical location; this labeling convention simply serves to reduce the number of node numbers. Next, each item $i \in I$ defines a node representing the location of the item. Finally, multiple nodes are utilized to represent the single packing station (depot). Although there is one *physical* packing station, our network representation requires the creation of multiple *replicas* (copies) of this station. Each replica is given a unique number, and is associated with exactly one robot. These replicas are required because each individual robot may visit the packing station multiple times; delivery (Freight) robots may visit multiple times to deliver items or to recharge, while picker (Fetch) robots will only visit the packing station to recharge. Each time a robot visits the packing station it will be assigned to a different replica of the station. Specifically, we define Δ_v^* to be the set of packing station replicas for vehicle $v \in V$. Note that $\Delta_{v_1}^* \cap \Delta_{v_2}^* = \varnothing$ for all $v_1 \neq v_2$ (i.e., all of these nodes are unique). A pre-processing step is required to determine the number of replicas that should be created for each robot. Thus, the entire set of nodes is given by $N = 0 \cup I \cup_{v \in V} \Delta_v^*$.

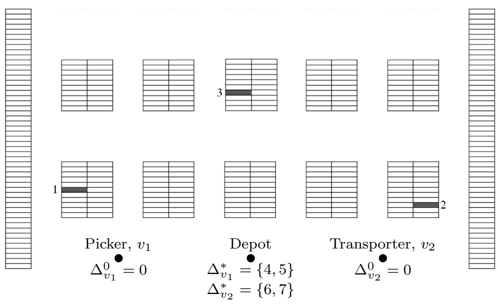
Additional notation characterizes the permissable travel movements of the robots. We define Δ_v^+ to be the set of nodes to which vehicle $v \in V$ may travel, such that $\Delta_v^+ \subseteq \{I \cup \Delta_v^*\}$. Note that $\Delta_v^0 \notin \Delta_v^+$ because a vehicle can never return to its initial location (vehicles may only *leave* the initial location). Furthermore, if an item associated with node $i \in I$ is too heavy for vehicle v, then $i \notin \Delta_v^+$. Next, given some node $j \in \Delta_v^+$ for a particular vehicle $v \in V$, $\Delta_{v,j}^-$ represents the set of nodes that could be visited immediately prior to node j. Thus, a vehicle may travel directly from node $i \in \Delta_{v,j}^-$ to node $j \in \Delta_v^+$. If $j \in I$ (i.e., if j represents the location of an item), then $\Delta_{v,j}^-$ contains Δ_v^0 (the vehicle's initial location), $I \setminus j$ (all other item locations), and Δ_v^* (all packing station replicas). However, if $j \in \Delta_v^*$ (i.e., if j is one of the packing station replicas), then $\Delta_{v,j}^-$ contains Δ_v^0 (this would mean that the vehicle travels directly from its initial location to a packing station), I (all item locations), and $\max\{\Delta_v^* < j\}$ (the largest replica node for vehicle v that is smaller than replica node j). Under this construction, a robot may move from one of its replica packing stations to another. However, it may only move to the next larger replica node. If a vehicle moves from replica to replica in an optimal solution, this is indicative of excess replicas being defined for this vehicle.

An example of the network structure is shown in Figure 2. There is one picker ($v_1 \in P$) and one transporter ($v_2 \in T$) which are to retrieve three items ($I = \{1, 2, 3\}$). Two depot replicas have been pre-defined for each vehicle ($\Delta_{v_1}^* = \{4, 5\}$ and $\Delta_{v_2}^* = \{6, 7\}$). Thus, $\Delta_{v_1}^+ = \{1, 2, 3, 4, 5\}$ and $\Delta_{v_2}^+ = \{1, 2, 3, 6, 7\}$ represent all of the nodes to which vehicles v_1 and v_2 may travel, respectively. If a checkmark or the vehicle's number appears in a column of the table in 2b, the node corresponding to that column is an element of Δ_v^+ for some vehicle v. For a given destination node, j, the rows of this column indicate if the origin node i is in the set $\Delta_{v,j}^-$. Using node 3 as an example destination node, for vehicle v_1, the set of nodes that may be visited immediately prior to node 3 includes 0 (the origin), 1, 2 (other item storage locations), and 4 (any of the depot replicas for v_1 except the last replica). That is, $\Delta_{v_1,3}^- = \{0, 1, 2, 4\}$. Similarly, $\Delta_{v_2,3}^- = \{0, 1, 2, 6\}$.

We define $\tau_{v,i,j}$ to be the time required for vehicle $v \in V$ to travel to node $j \in \Delta_v^+$ from node $i \in \Delta_{v,j}^-$. This parameter may include any additional travel time required by vehicles when turning corners. Note that $\Delta_v^0 = 0$ for all $v \in V$, but the travel time from Δ_v^0 to any location j, $\tau_{v,0,j}$, will incorporate the actual (potentially unique) initial location of vehicle v. Thus, $\tau_{v_1,0,j}$ does not necessarily equal $\tau_{v_2,0,j}$ for all $v_1, v_2 \in V$.

3.2 *Decision variables*

A variety of decision variables are employed in this coordinated vehicle routing problem. First, binary decision variable $x_{v,i,j}$ equals one if picker vehicle $v \in P$ travels from node $i \in \Delta_{v,j}^-$ to node



(a) An example warehouse layout showing three pick locations ($I = \{1, 2, 3\}$), a picker and a transporter. Both the picker (v_1) and transporter (v_2) start at node 0, although the vehicles are in different physical locations.

	To ($j \in \Delta^+_v$)							
		I			$\Delta^*_{v_1}$		$\Delta^*_{v_2}$	
From ($i \in \Delta^-_{v,j}$)		1	2	3	4	5	6	7
$\Delta^0_{v_1}, \Delta^0_{v_2}$	0	✓	✓	✓	v_1		v_2	
I	1		✓	✓	v_1	v_1	v_2	v_2
	2	✓		✓	v_1	v_1	v_2	v_2
	3	✓	✓		v_1	v_1	v_2	v_2
$\Delta^*_{v_1}$	4	v_1	v_1	v_1		v_1		
	5							
$\Delta^*_{v_2}$	6	v_2	v_2	v_2				v_2
	7							

(b) From/to combinations that are valid for both vehicles are indicated by a ✓. A particular vehicle number indicates that only that vehicle is allowed to travel from one node to another. For example, picker vehicle v_1 cannot travel to either of the depot replicas associated with vehicle v_2 (nodes 6 and 7). Vehicle v_1 may also not depart from node 5; only two depot replicas were created for this vehicle (nodes 4 and 5), and the last replica is terminal.

Figure 2.: An example network structure for a system with two vehicles and three pick locations.

$j \in \Delta^+_v$. For delivery vehicle $v \in D$, binary decision variable $y_{v,i,j}$ is similarly defined.

Coordination among the vehicles is a key component of this problem. It is assumed that items retrieved by a picker robot must be placed into a transport robot before the picker can proceed to the next item. Thus, an item hand-off can only occur at the location where the item was retrieved. Continuous decision variable $t_{v,j} \geq 0$ determines the time at which vehicle $v \in V$ arrives at node $j \in \Delta^+_v$ and is ready to conduct an activity at that node. For picker vehicles ($v \in P$), this time represents the earliest possible arrival to node j. For delivery vehicles ($v \in D$) receiving an item from a picker, the definition is nuanced. Here, $t_{v,j}$ represents the time at which the delivery vehicle may begin to receive item j. That is, the delivery vehicle is assumed to arrive at node $j \in I$ no earlier than the time at which the picker has actually retrieved the item. The binary decision variable $a_{v_1,v_2,j}$ establishes the pairing between a picker robot and a delivery robot at a particular item location, such that $a_{v_1,v_2,j} = 1$ if $v_1 \in P$ and $v_2 \in D$ are assigned to retrieve item $j \in I$.

Although picker and delivery vehicles are capacity constrained, only delivery vehicles may move while carrying an item. Continuous decision variable $w_{v,i,j} \geq 0$ represents the total weight of items carried by delivery vehicle $v \in D$ after leaving node j, having traveled from node i. Thus, if v travels from i to j, $w_{v,i,j}$ will include all weight loaded through location i plus the weight added at location j. Payload capacity limitations for *picker* vehicles are addressed in the definition of Δ^+_v, which prohibits a picker from visiting a location associated with an item that exceeds its capacity.

Three types of decision variables are associated with activities that occur at the depot. First, delivery vehicles form a queue when arriving at the packing station as they wait for their totes to be emptied. To monitor the order in which these vehicles arrive at the depot, binary decision variable $q_{j_1,j_2} = 1$ if vehicle $v_1 \in D$ arrives at its depot replica $j_1 \in \Delta^*_{v_1}$ before v_2 arrives at its depot replica $j_2 \in \Delta^*_{v_2}$. The battery-powered vehicles require periodic re-charging, which is performed at stations adjacent to the depot. The charge remaining on vehicle $v \in V$ when it arrives at depot replica $j \in \Delta^*_v \cup \Delta^0_v$ is given by continuous decision variable $0 \leq c_{v,j} \leq 1$. Note that the value of c_{v,Δ^0_v} is hard-coded to equal c'_v at the initial location. Third, $g_{v,j} \geq 0$ represents the duration that vehicle $v \in V$ spends charging at depot replica $j \in \Delta^*_v \cup \Delta^0_v$.

Finally, the makespan, which is to be minimized, is represented by continuous decision variable $m \geq 0$. Table 2 summarizes the decision variable notations of the PPT-VRP.

Table 2.: Summary of decision variable notation.

$a_{v_1,v_2,i} \in \{0,1\}$	$a_{v_1,v_2,j} = 1$ if $v_1 \in P$ and $v_2 \in D$ are assigned to retrieve item $j \in I$.
$0 \leq c_{v,j} \leq 1$	The charge remaining on vehicle $v \in V$ when it arrives at depot replica $j \in \Delta_v^* \cup \Delta_v^0$.
$g_{v,j} \geq 0$	The duration that vehicle $v \in V$ spends charging at depot replica $j \in \Delta_v^* \cup \Delta_v^0$.
$m \geq 0$	The required makespan to retrieve a pick list of items.
$q_{j_1,j_2} \in \{0,1\}$	$q_{j_1,j_2} = 1$ if vehicle $v_1 \in D$ arrives at its depot replica $j_1 \in \Delta_{v_1}^*$ before v_2 arrives at its depot replica $j_2 \in \Delta_{v_2}^*$.
$t_{v,j} \geq 0$	The arrival time at node $j \in \Delta_v^+$ for vehicle $v \in V$.
$w_{v,i,j} \geq 0$	The total weight of items carried by transporter $v \in D$ after leaving node j, having traveled from node i.
$x_{v,i,j}$, $y_{v,i,j} \in \{0,1\}$	$x_{v,i,j} = 1$ ($y_{v,i,j} = 1$) if picker $v \in P$ (transporter $v \in D$) travels from node $i \in \Delta_{v,j}^-$ to node $j \in \Delta_v^+$.

3.3 *MILP formulation*

The MILP formulation for the PPT-VRP is as follows.

$$
\begin{aligned}
\text{Min}\quad & m && (1)\\
\text{s.t.}\quad & m \geq t_{v,j} \quad \forall\, v \in D, j \in \Delta_v^*, && (2)\\
& \sum_{v\in P}\sum_{i\in\Delta_{v,j}^-} x_{v,i,j} = 1 \quad \forall\, j \in I, && (3)\\
& \sum_{v\in D}\sum_{i\in\Delta_{v,j}^-} y_{v,i,j} = 1 \quad \forall\, j \in I, && (4)\\
& 2a_{v_1,v_2,j} \leq \sum_{i\in\Delta_{v_1,j}^-} x_{v_1,i,j} + \sum_{i\in\Delta_{v_2,j}^-} y_{v_2,i,j} \quad \forall\, j \in I, v_1 \in P, v_2 \in D, && (5)\\
& a_{v_1,v_2,j} + 1 \geq \sum_{i\in\Delta_{v_1,j}^-} x_{v_1,i,j} + \sum_{i\in\Delta_{v_2,j}^-} y_{v_2,i,j} \quad \forall\, j \in I, v_1 \in P, v_2 \in D, && (6)\\
& \sum_{v_1\in P}\sum_{v_2\in D} a_{v_1,v_2,j} = 1 \quad \forall\, j \in I, && (7)\\
& t_{v_2,j} \geq t_{v_1,j} + s_{v_1,j}^{\text{pick}} - M(1 - a_{v_1,v_2,j}) \quad \forall\, v_1 \in P, v_2 \in D, j \in I, && (8)\\
& t_{v_1,j} \geq t_{v_2,i} + s_{v_1,i}^{\text{place}} + \tau_{v_1,i,j} - M(2 - a_{v_1,v_2,i} - x_{v_1,i,j}) \\
& \qquad \forall\, v_1 \in P, v_2 \in D, i \in I, j \in \{\Delta_{v_1}^+ : i \in \Delta_{v_1,j}^-\}, && (9)\\
& \sum_{j\in\{\Delta_v^+:\Delta_v^0\in\Delta_{v,j}^-\}} x_{v,\Delta_v^0,j} = 1 \quad \forall\, v \in P, && (10)\\
& \sum_{i\in\Delta_{v,j}^-} x_{v,i,j} = 1 \quad \forall\, v \in P, j \in \Delta_v^*, && (11)\\
& \sum_{i\in\Delta_{v,j}^-} x_{v,i,j} = \sum_{k\in\{\Delta_v^+:j\in\Delta_{v,k}^-\}} x_{v,j,k} \quad \forall\, v \in P, j \in \Delta_v^+, && (12)
\end{aligned}
$$

$$\sum_{i \in \Delta^-_{v,j}} x_{v,i,j} \leq 1 \quad \forall\ v \in P, j \in \Delta^+_v, \tag{13}$$

$$\sum_{j \in \{\Delta^+_v : i \in \Delta^-_{v,j}\}} x_{v,i,j} \leq 1 \quad \forall\ v \in P, i \in \{\Delta^+_v \cup \Delta^0_v\}, \tag{14}$$

$$\sum_{j \in \{\Delta^+_v : \Delta^0_v \in \Delta^-_{v,j}\}} y_{v,\Delta^0_v,j} = 1 \quad \forall\ v \in D, \tag{15}$$

$$\sum_{i \in \Delta^-_{v,j}} y_{v,i,j} = 1 \quad \forall\ v \in D, j \in \Delta^*_v, \tag{16}$$

$$\sum_{i \in \Delta^-_{v,j}} y_{v,i,j} = \sum_{k \in \{\Delta^+_v : j \in \Delta^-_{v,k}\}} y_{v,j,k} \quad \forall\ v \in D, j \in \Delta^+_v, \tag{17}$$

$$\sum_{i \in \Delta^-_{v,j}} y_{v,i,j} \leq 1 \quad \forall\ v \in D, j \in \Delta^+_v, \tag{18}$$

$$\sum_{j \in \{\Delta^+_v : i \in \Delta^-_{v,j}\}} y_{v,i,j} \leq 1 \quad \forall\ v \in D, i \in \{\Delta^+_v \cup \Delta^0_v\}, \tag{19}$$

$$t_{v,0} = 0 \quad \forall\ v \in V, \tag{20}$$

$$t_{v,j} \geq t_{v,i} + \left(s^{\text{pick}}_{v,i} + s^{\text{place}}_{v,i} + \tau_{v,i,j}\right) x_{v,i,j} - M(1 - x_{v,i,j})$$
$$\forall\ v \in P, j \in \Delta^+_v, i \in \{\Delta^-_{v,j} \cap I\}, \tag{21}$$

$$t_{v,j} \geq t_{v,i} + \sum_{v' \in P} s^{\text{place}}_{v',i} a_{v',v,i} + \tau_{v,i,j} - M(1 - y_{v,i,j}) \quad \forall\ v \in D, j \in \Delta^+_v, i \in \{\Delta^-_{v,j} \cap I\}, \tag{22}$$

$$t_{v,j} \geq t_{v,i} + g_{v,i} + \tau_{v,i,j} x_{v,i,j} - M(1 - x_{v,i,j})$$
$$\forall\ v \in P, j \in \Delta^+_v, i \in \{\Delta^*_v \cup 0 : i \in \Delta^-_{v,j}\}, \tag{23}$$

$$t_{v,j} \geq t_{v,i} + g_{v,i} + \tau_{v,i,j} y_{v,i,j} - M(1 - y_{v,i,j})$$
$$\forall\ v \in D, j \in \Delta^+_v, i \in \{\Delta^*_v \cup 0 : i \in \Delta^-_{v,j}\}, \tag{24}$$

$$t_{v,j_2} \geq t_{v,j_1} + s^{\text{drop}}_v \left(\sum_{i \in \{\Delta^-_{v,j_1} \setminus (\Delta^*_v \cap j_2)\}} y_{v,i,j_1} \right) + \tau_{v,j_1,j_2} y_{v,j_1,j_2} - M(1 - y_{v,j_1,j_2})$$
$$\forall\ v \in D, j_1 \in \{\Delta^*_v \setminus \max\{\Delta^*_v\}\}, j_2 \in \{\Delta^+_v \setminus j_1\}, \tag{25}$$

$$w_{v,i,j} \leq \hat{w}_v y_{v,i,j} \quad \forall\ v \in D, j \in \Delta^+_v, i \in \Delta^-_{v,j}, \tag{26}$$

$$w_{v,\Delta^0_v,j} = (w'_v + \bar{w}_j) y_{v,\Delta^0_v,j} \quad \forall\ v \in D, j \in \{I \cap \Delta^+_v\}, \tag{27}$$

$$w_{v,i,j} = \bar{w}_j y_{v,i,j} \quad \forall\ v \in D, j \in I, i \in \{\Delta^-_{v,j} \cap \Delta^*_v\}, \tag{28}$$

$$w_{v,j,k} \geq \sum_{\substack{i \in \Delta^-_{v,j} \\ i \neq k}} w_{v,i,j} + \bar{w}_k y_{v,j,k} - \hat{w}_v (1 - y_{v,j,k}) \quad \forall\ v \in D, k \in I, j \in \{I : k \neq j\}, \tag{29}$$

$$g_{v,0} = 0 \quad \forall\ v \in V, \tag{30}$$

$$c_{v,\Delta^0_v} = c'_v \quad \forall\ v \in V, \tag{31}$$

$$c_{v,j} \leq c_{v,i} + r_v g_{v,i} - d_v \left(t_{v,j} - (t_{v,i} + g_{v,i})\right)$$
$$\forall\ v \in V, j \in \Delta^*_v, i = \{\Delta^*_v \cup \Delta^0_v : i \in \Delta^-_{v,j}\}, \tag{32}$$

$$t_{v_2,j_2} \geq t_{v_1,j_1} + s^{\text{drop}}_{v_1}\left(\sum_{i \in \{\Delta^-_{v_1,j_1} \setminus \Delta^*_{v_1}\}} y_{v_1,i,j_1}\right) - M(1 - q_{j_1,j_2})$$

$$\forall\ v_1 \in D, v_2 \in \{D \setminus v_1\}, j_1 \in \Delta^*_{v_1}, j_2 \in \Delta^*_{v_2}, \tag{33}$$

$$q_{j_1,j_2} + q_{j_2,j_1} = 1 \quad \forall\ v_1 \in D, v_2 \in \{D : v_2 > v_1\}, j_1 \in \Delta^*_{v_1}, j_2 \in \Delta^*_{v_2}, \tag{34}$$

$$q_{j_1,j_2} = 1 \quad \forall\ v \in D, j_1 \in \Delta^*_v, j_2 \in \{\Delta^*_v : j_2 > j_1\}, \tag{35}$$

$$m \geq 0, \tag{36}$$

$$q_{j_1,j_2} \in \{0,1\} \quad \forall\ v_1 \in D, v_2 \in \{D \setminus v_1\}, j_1 \in \Delta^*_{v_1}, j_2 \in \Delta^*_{v_2}, \tag{37}$$

$$t_{v,j} \geq 0 \quad \forall\ v \in V, j \in \Delta^+_v, \tag{38}$$

$$w_{v,i,j} \geq 0 \quad \forall\ v \in D, j \in \Delta^+_v, i \in \Delta^-_{v,j}, \tag{39}$$

$$y_{v,i,j} \in \{0,1\} \quad \forall\ v \in D, j \in \Delta^+_v, i \in \Delta^-_{v,j}, \tag{40}$$

$$x_{v,i,j} \in \{0,1\} \quad \forall\ v \in P, j \in \Delta^+_v, i \in \Delta^-_{v,j}, \tag{41}$$

$$a_{v_1,v_2,j} \in \{0,1\} \quad \forall\ v_1 \in P, v_2 \in D, j \in I, \tag{42}$$

$$0 \leq c_{v,j} \leq 1 \quad \forall\ v \in V, j \in \{\Delta^*_v \cup \Delta^0_v\}, \tag{43}$$

$$0 \leq g_{v,j} \quad \forall\ v \in V, j \in \Delta^*_v. \tag{44}$$

The objective function (1) seeks to minimize the latest time at which all items are delivered to the packing station (depot), as limited by Constraint (2). Constraints (3) and (4) ensure that each item is retrieved by a picker vehicle and placed into a delivery vehicle. Each item location must be visited by both a picker and a transporter.

Constraints (5)–(9) coordinate the picker and delivery vehicles at each item location, where Constraints (5), (6), and (7) set the appropriate value of $a_{v_1,v_2,j}$ to pair a picker with a transporter, while Constraints (8) and (9) establish the timing of this coordination. Conversely, Constraint (6) sets $a_{v_1,v_2,j} = 1$ if v_1 and v_2 meet at j. Constraint (7) ensures that each item is associated with exactly one picker/transporter pair. Next, Constraint (8) specifies that a picker may retrieve an item before a transporter arrives, but the transporter is not deemed to arrive at this location until the picker has completed the picking operation. Constraint (9) prohibits a picker vehicle from moving to the next location, j, until the placement of an item at i is completed.

The value of M, which represents a sufficiently large number, corresponds to an upper bound on the makespan. One valid bound may be calculated as the maximum cumulative time required to visit all nodes by a single vehicle, such that $M = \max\{\tau^{\text{upper}}_v + \tau^{\text{charging}}_v\}$, where

$$\tau^{\text{upper}}_v = \tau_{v,\Delta^0_v,\text{depot}} + \sum_{j \in I} 2\tau_{v,\text{depot},j} + \max\left\{\sum_{j \in I}\left(s^{\text{pick}}_{v',j} + s^{\text{place}}_{v',j}\right)\right\} + |I| \max\left\{s^{\text{drop}}_{v''}\right\}$$

for all $v \in V, v' \in P$, and $v'' \in D$. Here, τ^{upper}_v represents the time for vehicle $v \in V$ to travel from its initial location to the depot, then to make round-trip visits from the depot to each picking location, plus the maximum service time for the pick, place, and drop activities. The value of τ^{charging}_v, which represents the required charging time for vehicle v to travel a route of duration τ^{upper}_v, is given by

$$\tau^{\text{charging}}_v = \frac{\max\{0, (d_v \tau^{\text{upper}}_v - c'_v)\}}{r_v}.$$

Valid vehicle routes are established by Constraints (10)–(14). Constraint (10) requires each picker to depart from its initial location, while Constraint (11) ensures that each picker tour ends at a depot replica. Conservation of flow for picker vehicles is guaranteed by Constraint (12). Constraints (13) and (14) prohibit pickers from visiting or leaving any node more than once, respectively. Constraints (15)–(19) are analogous to Constraints (10)–(14) for delivery vehicles.

Constraints (20)–(25) incorporate travel time into the routing process, where Constraint (20) initializes the start time for all vehicles to be zero. Constraint (21) states that, if a picker travels from i to j (where i is associated with picking up an item), then the arrival time to j cannot be before the arrival time to i plus the total service time at i plus the travel time from i to j. Similarly for delivery vehicles, Constraint (22) guarantees that a transporter's arrival time to j cannot be earlier than the summation of the arrival time to i, the placement service time performed by the partnering picker at i, and the travel time from i to j. Constraints (23) and (24) ensure valid start times when a picker or transporter leave a depot replica, respectively, while Constraint (25) captures the drop-off time required before visiting subsequent locations.

Payload limitations for delivery vehicles are addressed by Constraints (26)–(29). In (26), the total weight carried by delivery vehicle $v \in D$ after visiting node j cannot exceed the capacity limit. When a transporter leaves its initial location (Δ_v^0) and travels to some location j, the total carried weight equals the initial weight plus the quantity picked up at location j, as in Constraint (27). Constraint (28) determines the payload weight carried by transporter v when picking up the first item after leaving a depot replica. Constraint (29) forces the payload weight to be at least as large as the summation of weight when v leaves j, and the weight loaded at k.

Constraint (30) initializes the charging time at each vehicle's initial location to be zero. Similarly, (31) establishes the initial charge of vehicle v when leaving Δ_v^0. The left-hand side of (32) represents the charge of vehicle v when arriving at depot j. This charge cannot exceed the charge when it arrived at depot i (note that depot replicas are ordered such that i precedes j) plus the additional charge acquired while at station i minus the discharge that occurs between i and j.

Constraints (33), (34), and (35) address queueing of transport vehicles at the depot. Constraint (33) establishes the effective arrival time of a transporter at the depot, taking into account the arrival order of all transport vehicles. Decision variable $q_{j_1,j_2} = 1$ if v_1 arrives before v_2, where j_1 is the depot replica associated with v_1. The time that v_2 may begin service at the depot must not be before v_1 has completed. Constraint (34) considers two depot replica nodes that are used by different transport vehicles, ensuring that exactly one of the replica nodes is used before the other. Similarly, Constraint (35) hard-codes the values of q_{j_1,j_2} for a particular transport vehicle to force a given vehicle to utilize its replica nodes in order. The model concludes with decision variable definitions in Constraints (36)–(44).

3.4 *Modifying the model to account for humans*

While the above PPT-VRP model was formulated specifically for picker and transport robots, it is straightforward to modify it to address combinations of human pickers and robotic delivery vehicles. For these mixed modes, we consider only the case of a human performing picking operations and a delivery robot transporting picked items to the depot. Given that humans are currently more adept at identifying and picking items from a shelf, and that delivery robots are likely faster at moving material, it would seem impractical to replace the delivery robot with a human. Thus, to replace the picker robot with a human (i.e., to model a follow-pick system), we consider the set P to represent all human pickers (rather than picker robots). It is then sufficient to modify the parameter values describing travel time ($\tau_{v,i,j}$), payload capacity ($\hat{w}_v$), picking time ($s_{v,i}^{\text{pick}}$), and placing time ($s_{v,i}^{\text{place}}$) for all $v \in P$. Constraints (30) – (32), which govern battery consumption, may be safely ignored for all $v \in P$.

For the purposes of comparing the robot-only and hybrid human-robot systems, it is also beneficial to determine the optimal routing assignments associated with traditional human-based order picking. In this scheme, each human worker moves through the warehouse with a cart and performs both picking and transporting operations. As in the PPT-VRP, the routing constraints prohibit any item location from being visited by more than one picker. Additionally, we assume that a queue may still be formed at the packing station. However, no charging time is required for the human. To re-use the framework of the PPT-VRP model, we let $V = P = D$ be redefined as the set of human pickers, and let decision variable $y_{v,i,j} = 1$ be redefined to indicate that human $v \in V$ should travel from location $i \in \Delta^-_{v,j}$ to location $j \in \Delta^+_v$. The human routing model presented below incorporates Constraint (46), which is modified from Constraint (22) to ensure that every human picker only leaves a picking location after performing both item picking and placing. Thus, the human-only problem becomes a VRP with the addition of packing station queueing.

$$\text{Min} \quad m \tag{45}$$

$$\text{s.t.} \quad t_{v,j} \geq t_{v,i} + \left(s^{\text{pick}}_{v,i} + s^{\text{place}}_{v,i} + \tau_{v,i,j}\right) y_{v,i,j} - M(1 - y_{v,i,j})$$
$$\forall\, v \in D, j \in \Delta^+_v, i \in \{\Delta^-_{v,j} \cap I\}, \tag{46}$$
$$\text{Constraints (2), (4), (15) – (20), (25) – (29), (33) – (40).}$$

4. Numerical analysis

A series of numerical studies was conducted to (1) assess the impact of warehouse layout configurations on the performance of PPT-VRP systems, and (2) quantify the relative impacts of adding picker or transporter robots, increasing picker speeds, or increasing transporter capacities. All computational work was conducted on a PC with an Intel i5-2410m processor and 12 GB RAM running Microsoft Windows 8 in 64-bit mode. The PPT-VRP models were solved by Gurobi 6.0.3 (Gurobi Optimization 2016) via Python version 2.7.5.

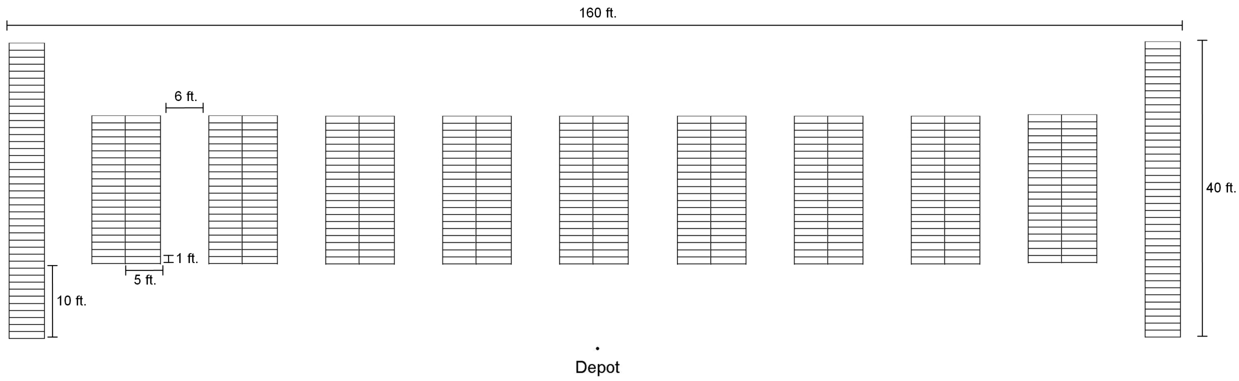
4.1 *Test instance creation*

Eighteen different warehouse layouts were generated, each of which is characterized by the number of vertical picking aisles (PAs), the number of horizontal cross aisles (CAs), and the location of the depot. Specifically, these test layouts feature either 2, 6, or 10 PAs; 2, 3, or 4 CAs; and either traditional or centrally-located depots. Traditional depots (TDs) are typically located at the horizontal midpoint along the lower boundary of the warehouse, while less-common central depots (CDs) are located in the middle of the warehouse.

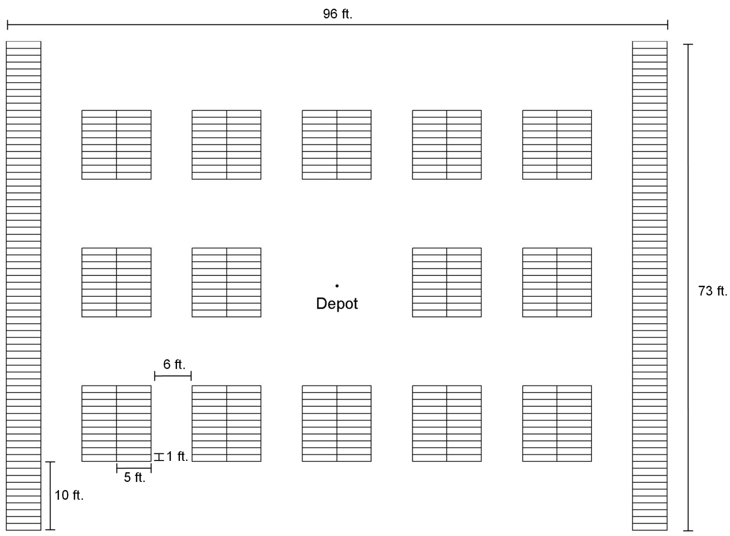
Consistent with the human-based warehousing literature (e.g., Pan, Wu, and Chang (2014)), we consider CA widths of 10 feet, PA widths of 6 feet, and storage racks with footprints of 1-foot wide by 5-feet deep. Because both the Fetch and Freight robots have bases of 22-inches in diameter (Wise et al. 2016), three robots can occupy a PA side-by-side with room to spare. Thus, aisle blocking is not considered in the numerical analysis.

Each layout contains 460 ± 20 picking locations, with slight variations owing to the loss of picking locations surrounding CDs. Furthermore, changes in the numbers of PAs and CAs affect the quantity of storage locations. For example, as noted by Roodbergen and de Koster (2001a), adding CAs increases the space requirements of a warehouse. Two of the generated warehouse layouts are illustrated in Figure 3.

Three metrics are employed to quantify the differences among the generated layouts. These include the aspect ratio (α), the average distance from the depot to each picking location (ADFD), and the average distance between picking locations (ADBPL). Table 3 provides a summary, along with the total space requirements and the number of storage locations. The ADFD and ADBPL metrics have been widely used to estimate the travel distances of different warehouse picker routing



(a) A warehouse with a traditional depot, 10 PAs, and 2 CAs. There are 440 storage locations in this 595-square-meter facility, identified as Layout 7 in Table 3.



(b) A warehouse with a central depot, 6 PAs and 4 CAs. There are 454 storage locations in this 651-square-meter facility, identified as Layout 15 in Table 3.

Figure 3.: Illustrations of two generated layouts.

policies (e.g., Ozturkoglu, Gue, and Meller (2012)). Note that layouts with traditional depots have a higher ADFD, but a slightly lower ADBPL, than layouts with centrally-located depots.

For each layout, 300 randomly-generated 5-item pick lists were created, resulting in 5,400 test instances. The average computational time for solving a 5-item problem to optimality was 1.9 seconds, with a maximum time of 190 seconds. A uniform storage policy is applied, as per previous batch order picking research (e.g., Roodbergen, Vis, and Taylor (2015)). The use of fixed pick list sizes is common in the order picking literature (e.g., Pan, Wu, and Chang (2014); Shqair, Altarazi, and Al-Shihabi (2014)). However, while some studies considered pick list sizes ranging from 4 to 80 items, the complexity of the PPT-VRP would require excessive computational time to obtain provably optimal solutions for larger pick lists. Note that the PPT-VRP – which extends the classical VRP to include heterogeneous vehicles, vehicle coordination constraints, and a min-max objective function – is an NP-hard problem, as it includes the single-vehicle TSP as a special case. Although employing heuristic methods would provide solutions to larger-scale problems, these would not be provably optimal. As such, any analysis regarding the impacts of layout configurations would be subject to an uncertain optimality gap.

Picker robots travel at a speed of 1.0 m/s, while transport robots travel at 2.0 m/s (Wise et al.

Table 3.: A summary of warehouse designs generated for the numerical analysis. Layout 7 is shown in Figure 3a, while Layout 15 is shown in Figure 3b.

	Layout Parameters			Layout Attributes				
Layout ID	Depot Type	# of PAs	# of CAs	α	ADFD [m]	ADBPL [m]	# of Storage Locations	Space Req. [m^2]
1	TD	2	2	0.26	22.82	19.90	456	368.64
2		2	3	0.25	23.74	16.65	460	386.48
3		2	4	0.24	24.66	16.53	464	404.31
4		6	2	1.78	17.48	18.50	448	481.61
5		6	3	1.52	19.10	17.09	456	561.88
6		6	4	1.32	20.71	17.62	464	642.15
7		10	2	4	20.67	23.40	440	594.58
8		10	3	3.2	22.58	22.89	460	743.22
9		10	4	2.67	22.89	23.84	480	891.87
10	CD	2	2	0.25	12.07	20.82	456	386.48
11		2	3	0.24	12.19	16.89	458	389.45
12		2	4	0.23	13.32	17.50	464	422.15
13		6	2	1.71	16.70	19.01	448	499.45
14		6	3	1.52	12.22	17.18	450	561.88
15		6	4	1.32	14.37	18.04	454	651.06
16		10	2	3.81	20.49	24.08	456	624.31
17		10	3	3.2	16.77	22.98	456	743.22
18		10	4	2.67	19.00	24.14	466	891.87

2016). Each picker robot is assumed to require 5-seconds to pick up or place an item into a tote. Humans are assumed to travel at 0.6 m/s with a cart, and 1 m/s without; humans require 1.5 seconds to pick up and place an item into a cart (Yu and de Koster 2010).

The tote drop-off time is assumed to be 5 seconds for either transport robots or humans. The capacity of each transporter is varied, such that each may hold 1, 3, or 5 items in a tote. This allows cases where a transporter with relatively low capacity must revisit the depot to fulfill a pick list. No capacity limitations are placed on human-based operations. The experimental parameters are summarized in Table 4.

Table 4.: A summary of the experimental parameter setting.

Parameter	**Levels**
Number of CAs	2, 3, 4
Number of PAs	2, 6, 10
Depot Type	CD, TD
Number of Pickers	1, 2, 3
Number of Transporters	1, 2, 3
Transporter Payload Capacity	Low (1 item), Medium ($\lceil \|I\|/2 \rceil$), High ($\|I\|$)

4.2 *Relationships between layout designs and robot fleet composition*

Three order-picking systems are evaluated to determine the degree to which warehouse layouts impact the relative performance of robotic-based picker systems against traditional human-based picking operations. The first is based on the Fetch and Freight picker and transporter robots, denoted as F&F. The second combines human pickers with robotic transports, denoted as H&F (or human and Freight). In this follow-pick collaboration, humans perform the tasks of picking and placing items, while the robotic transporter performs delivery operations. The H&F approach leverages the shorter item retrieval times for humans versus picker robots and the faster travel speeds for transporter vehicles versus humans. Nine combinations of pickers and transporters are considered in the F&F and H&F systems, from one picker and one transporter (1/1), to three pickers and three transporters (3/3). Finally, the third system considers a single human in a traditional picking system, which serves as a baseline of comparison against the F&F and H&F systems.

In the following analysis, "percentage improvement" refers to the improvement in makespan (i.e., decision variable m) relative to that of a traditional human-based system with a single worker. It is important to note that the *absolute* efficiency improvements are not of significant value, since, for example, it is clear that the case of three pickers and three transporters is expected to be more efficient than the single human case. Instead, the focus is on the comparison of *relative* percentage improvements between the different factor combinations (e.g., comparing 2/2 to 3/1).

There are numerous interaction effects among warehouse layout properties and the robot fleet composition. While the following study did not consider a full factorial design of experiments or the use of ANOVA, the focus is on the overall trends and relationships among key problem parameters.

4.2.1 Impacts of PAs, CAs, and depot location

Table 5 highlights the performance efficiency impacts associated with changing the numbers of cross aisles and picking aisles. This table reveals that additional PAs lead to efficiency improvements. While neither the ADBPL nor the ADFD exhibit a consistent trend as the number of PAs increase (both metrics decrease for 6 PAs and then increase for 10 PAs), note that the ratio of ADBPL to ADFD increases. Conversely, as the number of CAs increases, the ADBPL:ADFD ratio decreases. This corresponds to a decrease in the percentage improvement over the single-human system. This is consistent with Roodbergen and de Koster (2001a) in the context of human-only systems, who note that increasing the number of CAs might not improve performance despite creating more picking route options.

Table 5.: Percentage improvement over a single human picker for combinations of pick- and cross-aisles (as summarized in the northwest corner of the table). The corresponding ADBPL and ADFD are in units of meters, while the ratio of these distances is unitless.

		CAs						
		2	**3**	**4**	**Avg**	**ADBPL**	**ADFD**	**ADBPL:ADFD**
PAs	**2**	37.2	35.7	35.6	36.1	18.0	18.1	1.00
	6	40.4	41.1	39.5	40.4	17.9	16.8	1.07
	10	42.7	42.6	42.4	42.6	23.6	20.4	1.15
Avg		40.1	39.8	39.2				
ADBPL		21.0	18.9	19.6				
ADFD		18.4	17.8	19.2				
ADBPL:ADFD		1.14	1.07	1.02				

The impacts of traditional versus centrally-located depots are shown in Table 6, which reveals that CDs outperform TDs by 4.4% overall and by 7.9% when low-capacity transporters are employed. This table also shows that, with low-capacity transporters and a traditional depot, a 1-picker 1-transporter combination performs worse than a single human worker.

In Figure 4 the relationship between ADFD and depot location is highlighted. In particular, the higher ADFD associated with TDs makes the F&F system more competitive with H&F, as the longer travel distances for the transporter offset the faster item retrieval times of the picker.

Table 6.: Percentage improvement over a single human picker, as categorized by transporter capacity, depot type, and mix of picker/transporter quantities.

	Depot	**Picker/Transporter Combination**										
Cap.	**Type**	**1/1**	**1/2**	**1/3**	**2/1**	**2/2**	**2/3**	**3/1**	**3/2**	**3/3**		**Avg**
1	**TD**	-0.2	24.5	27.2	2.6	41.5	49.6	2.6	42.1	52.9		27.0
	CD	10.2	24.9	25.9	18.0	50.0	54.9	18.0	51.6	60.2		34.9
3	**TD**	24.0	27.5	27.5	38.0	51.0	51.0	39.8	56.1	56.5		41.3
	CD	25.1	26.1	26.1	43.4	55.7	55.7	46.1	62.5	63.2		44.9
5	**TD**	28.1	28.2	28.2	45.7	51.8	51.8	49.2	57.2	57.6		44.2
	CD	26.0	26.1	26.1	46.9	55.7	55.7	51.5	62.5	63.2		46.0
	TD Avg	17.3	26.7	27.6	28.8	48.1	50.8	30.5	51.8	55.6	37.5	
	CD Avg	20.5	25.7	26.0	36.1	53.8	55.4	38.5	58.9	62.2	41.9	

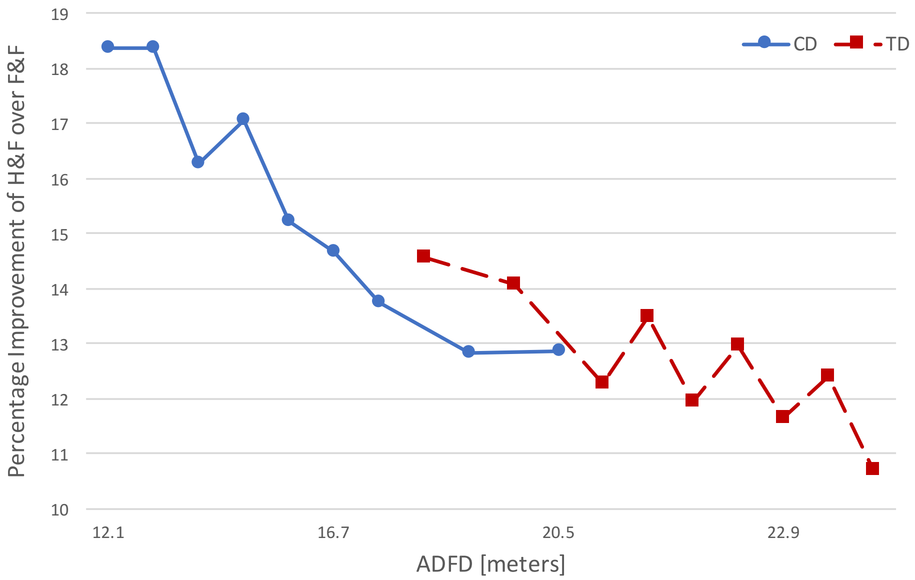


Figure 4.: As the ADFD increases, the relative performance of F&F to H&F has an improving trend. Note that TDs tend to have a larger ADFD than CDs.

4.2.2 Impacts of picker type and robot fleet properties

Table 7 shows the relationships among transporter capacity, picker type (Fetch versus human), and the picker/transporter mix. As previously observed, the H&F systems are more efficient than their F&F counterparts, as the human picker is assumed to grasp items faster than the robot. When transporter capacity is sufficient to hold all of the items on the pick list, there is no benefit associated with having more transporters than pickers. In such high-capacity cases, each transporter simply follows a picker during the item retrieval process. However, when transporter capacity is low, having more transporters than pickers is advantageous, as low-capacity transporters must make multiple visits to the packing station.

4.3 *Improving system efficiency by modifying the robot fleet*

We now turn our attention to the question of how best to modify the order picker assortment. Such an action would be relevant to a company considering changes to an existing fleet of robots.

Table 7.: Percentage improvement over a single human picker, as categorized by transporter capacity, picker type (Fetch vs. human), and mix of picker/transporter quantities.

	Picker	**Picker/Transporter Combination**									
Cap.	**Type**	**1/1**	**1/2**	**1/3**	**2/1**	**2/2**	**2/3**	**3/1**	**3/2**	**3/3**	**Avg**
1	**F**	-3.2	14.5	15.4	3.5	41.2	46.8	3.5	42.8	53.2	24.2
	H	13.2	34.9	37.8	17.1	50.4	57.7	17.1	50.9	59.9	37.7
3	**F**	13.7	15.4	15.4	34.1	47.3	47.3	36.8	55.3	55.9	35.7
	H	35.4	38.1	38.1	47.4	59.3	59.3	49.1	63.3	63.8	50.4
5	**F**	16.1	16.1	16.1	39.7	48.1	48.1	44.8	56.3	57.0	38.0
	H	38.1	38.1	38.1	52.9	59.3	59.3	55.9	63.4	63.8	52.1
	F Avg	8.9	15.4	15.6	25.8	45.5	47.4	28.4	51.5	55.4	32.6
	H Avg	28.9	37.0	38.0	39.1	56.4	58.8	40.7	59.2	62.5	46.7

In particular, the following analysis considers four types of fleet enhancements:

- Add a picker (denoted as +1P);
- Add a transporter (+1T);
- Increase transporter capacity (+2C), where capacities are 1, 3, or 5 units; or
- Upgrade picker grasp and place speeds to match human capabilities (F-to-H), which is equivalent to replacing a Fetch robot with a human picker.

As in the previous analysis, we limit the number of pickers and transporters to 3, and the capacity of transporters to 5. Thus, +1P (+1T) is not an option if 3 pickers (transporters) are already in the system, and +2C is not an option for transporters that already have a capacity of 5 units. Transporters with a capacity of 1, 3, or 5 items are denoted as low (L), medium (M), or high (H), respectively. Vehicle quantities are represented as the number of pickers / number of transporters / capacity of transporters. For example, 2/1/M represents a system with 2 pickers and 1 transporter with a medium (3-item) capacity.

ADFD and ADBPL are effective metrics for determining the fleet enhancements that lead to the greatest system performance improvements, as illustrated in Figure 5. Each plot is labeled with the initial quantities of picker and transporter robots. Colors in the plots indicate the enhancements producing the greatest efficiency improvements among all possible modifications and capacity levels. For example, the green area in the 1P/1T plot indicates that the efficiency improvement achieved by performing +1T in the 1/1/L situation is not only greater than the improvements realized by executing any of the other three actions in the 1/1/L scenario, but also greater than those resulting from implementing any of the four modifications in the 1/1/M and 1/1/H systems.

The vehicle combinations may be partitioned into four groups, allowing a categorization of the most beneficial fleet enhancements, as summarized in Table 8.

In the case of just one picker and one transporter (Group 1), the interplay between ADFD and ADBPL is important. When both ADFD and ADBPL are low, F-to-H yields the greatest improvement because the faster robot travel speeds provide less benefit for shorter travel distances. However, when the ADFD and ADBPL are both high, the transporter reaches the picking locations after the picker, since the influence of the ADFD on the transporters is greater than that of the ADBPL on the pickers. Thus, adding transporter capacity is preferred. When the ADFD is high but the ADBPL is low, adding a transporter helps offset the additional distance from the packing location. Conversely, when the ADBPL is high but the ADFD is low, adding a picker is preferable since the transporter tends to arrive at the picking locations before the picker, which must travel a longer distance between locations.

If the number of transporters exceeds the number of pickers (Group 2), ADBPL is the primary determinant for making fleet changes. When the ADBPL is low, faster picker grasping and placing

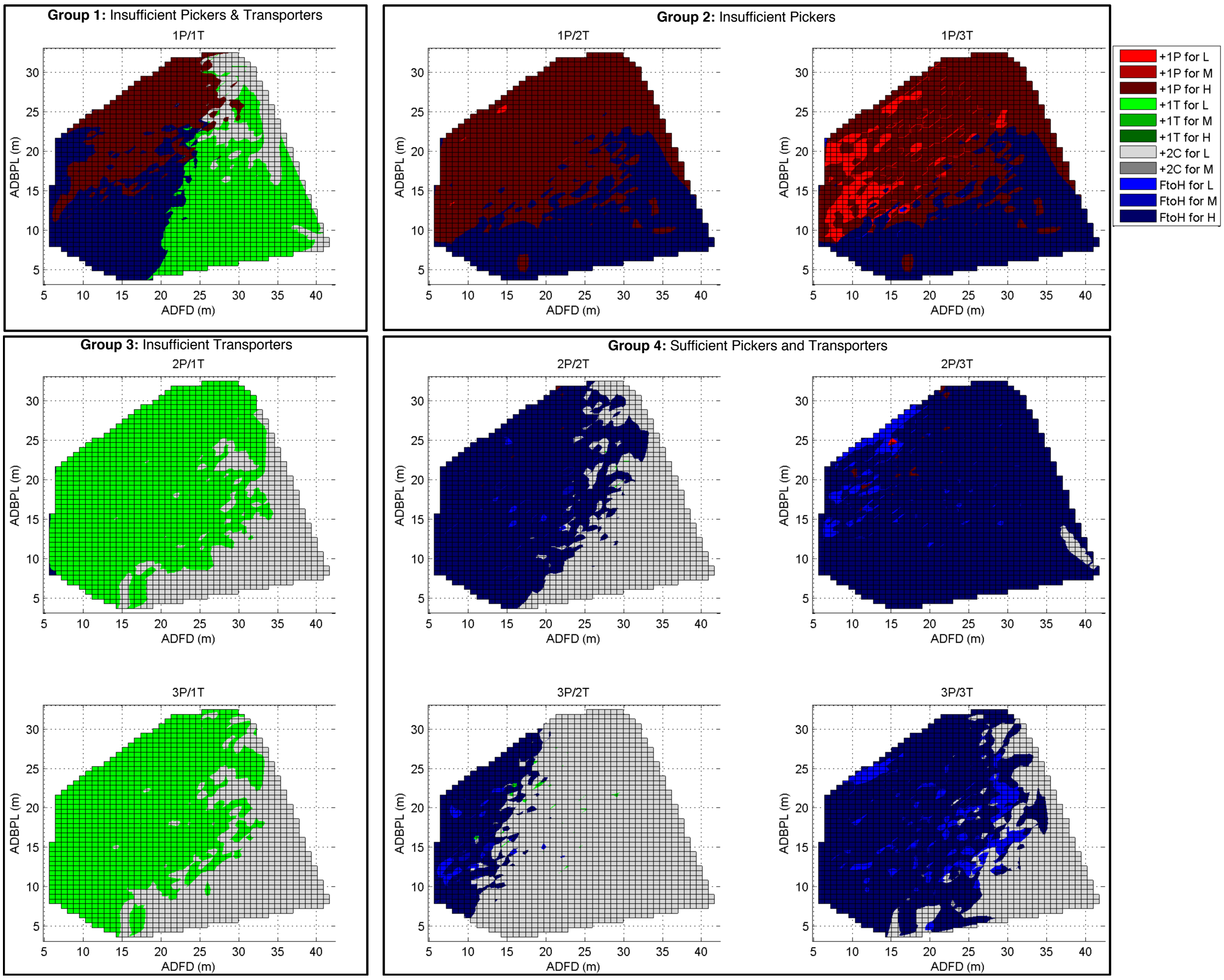


Figure 5.: Determining the most beneficial enhancements to a robot picker fleet via the ADFD and ADBPL metrics. Each subplot represents an initial combination of picker/transporter quantities. These subplots have been grouped according to combinations exhibiting similar characteristics.

Table 8.: Most beneficial fleet enhancement as a function of ADFD, ADBPL, and initial fleet composition.

		Group 1	**Group 2**	**Group 3**	**Group 4**
ADFD	**ADBPL**	1P/1T (Insuf. P & T)	1P/2T, 1P/3T (Insuf. P)	2P/1T, 3P/1T (Insuf. T)	2P/2T, 2P/3T, 3P/2T, 3P/3T (Suf. P & T)
Low	**Low**	F-to-H	F-to-H	+1T	F-to-H
Low	**High**	+1P	+1P	+1T	F-to-H
High	**Low**	+1T	F-to-H	+2C	+2C
High	**High**	+2C	+1P	+2C	+2C
Key Metric:		Both	ADBPL	ADFD	ADFD

speeds (F-to-H) are desirable. However, when ADBPL is high, the time savings associated with adding a picker (thus allowing the pickers to divide the workload) are greater than any speed improvements offered by F-to-H. As expected, adding transporters or transporter capacity is not beneficial for this group.

If the system has more pickers than transporters (Group 3), the ADFD dominates the decision-making process. Adding transporter capacity is most beneficial when ADFD is high, as this reduces the frequency with which the transporters must travel between picking locations and the depot. However, when ADFD is low, adding transporters is preferred (particularly if these transporters have low capacity relative to the pick list size).

ADFD is also the key metric in Group 4, which contains larger quantities of pickers and transporters than the other groups. As in Group 3, increasing transporter capacity is most beneficial when ADFD is high. However, improving picker capabilities (F-to-H) is preferable for low ADFD. Note that the threshold defining low vs. high ADFD is a function of the ratio of transporters to pickers; as this ratio increases, so to does the preference to enhance picker capabilities (i.e., F-to-H).

4.3.1 System efficiency improvement within larger warehouses

Two additional studies were conducted with larger pick lists in larger warehouses. The first study considered a warehouse with a TD, 6 CAs, 20 PAs, and dimensions of 36.58 by 97.54 meters. There are 2,520 storage locations in this 3,567-square-meter facility. For this layout, 300 randomly-generated 10- and 20-item pick lists were created under a uniform storage policy. Similar to the prior analysis, we investigate the makespan improvement associated with (1) adding a picker; (2) adding a transporter; (3) increasing the transporter capacity by 5 units for the 10-item problems, or by 10 units for the 20-item problems; and (4) increasing the pick/place speeds of the picker robot. The default payload capacity of the transporter is one-half of the pick list size (i.e., "medium" capacity). The case of a single picker and a single transporter serves as a baseline for comparison.

A total of 3,000 test problems were solved – 2 levels of pick list sizes, each associated with 300 pick lists and 5 vehicle configurations (the baseline plus the 4 modifications). The larger-sized pick lists require additional computational time for Gurobi. Therefore, we set a 10-minute cutoff time for the 10-item problems and a 20-minute cutoff time for the 20-item problems. The 10-item problems had an average optimality gap of 14.5%, while the 20-item problems had an average gap of 18.1%. Although these gaps are rather large, we re-ran several of the test instances with a 6-hour cutoff time and discovered that the incumbent solutions remained unchanged, but the lower bounds increased (indicative of weak lower bounds provided by the LP relaxation of the MILP formulation). Thus, these reported optimality gaps are likely inflated. We speculate that there are two primary reasons for these gaps to be larger than what is typically found with classical VRPs. First, the PPT-VRP uses a min-max objective, which likely results in the loose lower bound. Second, the PPT-VRP includes coordination constraints among the picker and transporter vehicles, which increases the difficulty of solving the problem within short cutoff times.

Figure 6 illustrates the preferred fleet modifications for the first study as a function of ADFD and ADBPL. For 10-item problems (left figure), it is generally most beneficial to add a picker (+1P). However, when ADFD is high and ADBPL is low, adding a transporter (+1T) and increasing capacity (+5C) are preferable. This is consistent with our previous findings for smaller warehouses with smaller pick lists. However, while the smaller warehouses also benefitted from improving picker capabilities (i.e., F-to-H), in the larger warehouses with 10 items faster pick/place times are dominated by the longer required travel distances.

Conversely, when considering 20-item pick lists in the larger warehouse (right plot of Figure 6), we see that F-to-H becomes more beneficial. We observe that the shaded region for 20-item problems is smaller than the 10-item problems, indicating the convergence of ADFD and ADBPL as more items are added to a pick list. The combination of larger pick lists and shorter travel distances places more importance on reducing the pick/place time. For this reason, we also note that adding a transporter becomes less beneficial in the 20-item problems.

The second study considered an even larger warehouse, with an area of 87,334 square meters (roughly 303 by 288 meters). This is similar in size to the recently-constructed one-million square foot warehouses of Amazon (Litten 2014) and Tesla (Kessler 2016). The warehouse in this second

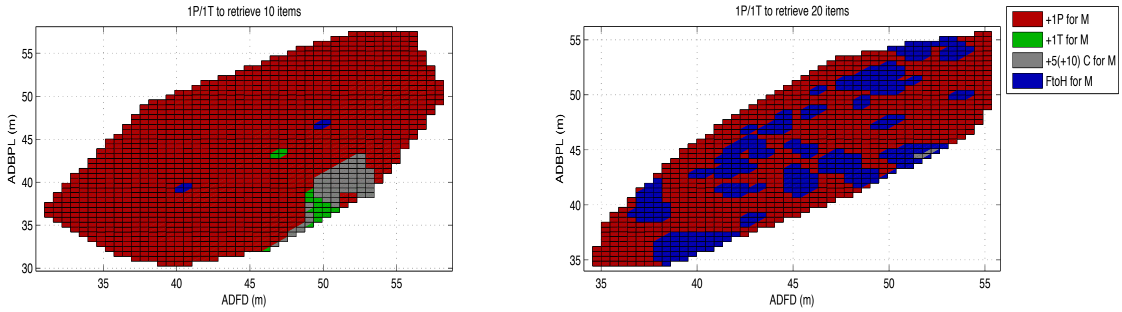


Figure 6.: Recommended fleet enhancements for the first case study with 10- and 20-item pick lists.

study has a TD, 3 CAs, 90 PAs, and 165,660 racks. We generate 40 10-item pick lists under a uniform storage policy.

Nine vehicle combinations (from 1 picker / 1 transporter to 3 pickers / 3 transporters) were considered, along with three levels of payload capacity (low of 1, medium of 5, and high of 10). Both the F&F and H&F systems were investigated, resulting in 2,160 test problems. These problems were solved via Gurobi, with a cutoff time of 10 minutes per problem. The average optimality gap was 12.6% (again, this large gap is likely due to the loose bound of the LP relaxation).

Figure 7 illustrates the most beneficial fleet modifications for the second study, in which travel distances are much longer due to the size of the warehouse. We note the dominance of adding capacity, particularly in the cases where the number of transporters is less than or equal to the number of pickers (i.e., 1P/1T, 2P/1T, 2P/2T, 3P/1T, 3P/2T, and 3P/3T).

For a baseline of 1P/1T, adding a transporter is most beneficial when ADFD and transporter capacity are low. However, adding capacity becomes a better choice when the ADFD increases; the size of the warehouse makes it costly for transporters to revisit the packing area (as is required when transporter capacity is low).

For the case of excess transporters (i.e., 1P/2T and 1P/3T), adding a picker provides the most impact in general. This is consistent with the previous analysis that found that F-to-H becomes less effective under larger warehouses. We also observe that adding capacity offers the most improvement for 1P/2T when the ADBPL is low. Furthermore, for 2P/3T with high payload capacity, adding a picker is preferred when both the ADFD and the ADBPL are low, which indicates that three high capacity transporters can support at least three pickers well under this situation. Otherwise, adding transporter capacity is desirable when the payload capacity is low.

This study with larger warehouses provides several managerial insights. First, increasing the speed of item pickup/place activities provides less impact under large scale warehouses with smaller pick lists, but becomes more important when the pick list size increases. Second, in larger warehouses, increasing the capacity of the transporters is more beneficial than adding more low-capacity transporters. This might be an attractive finding for managers, as adding transporter capacity (perhaps via larger totes) may be less expensive than purchasing additional transporters. Moreover, consistent with the previous findings, it is often preferable to have at least as many pickers as transporters, particularly if the transporters are of high capacity. Finally, the analysis again demonstrates that the performance of the order-picking system is dependent upon the ADFD and ADBPL metrics.

5. Conclusions and future research opportunities

This paper was motivated by the availability of specialized "picker" robots that can retrieve items from storage locations and "transport" robots that can bring these items to a packing station. Based on these capabilities, a new problem, the PPT-VRP, was defined to route these mobile

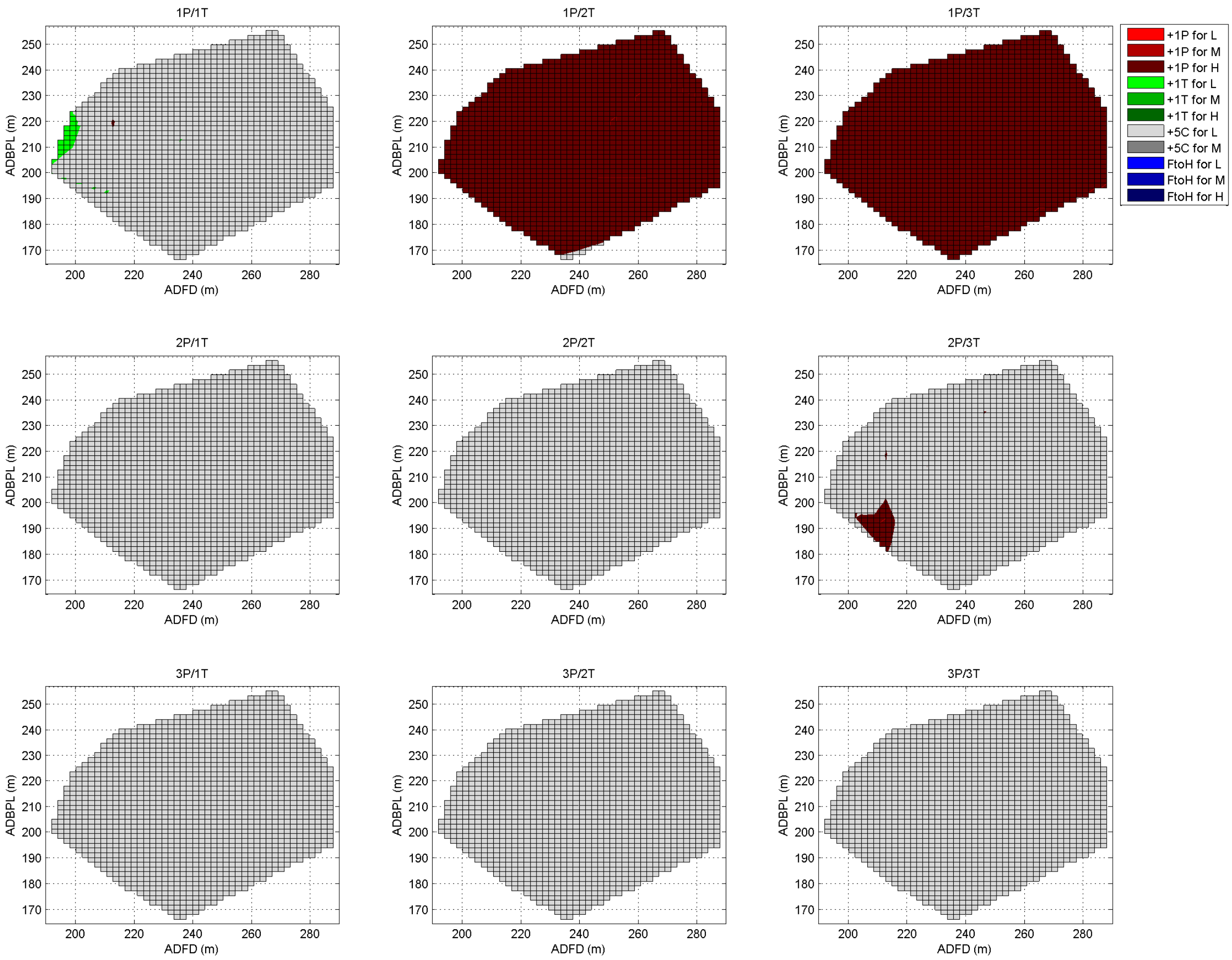


Figure 7.: Recommended fleet enhancements for the second case study with 10-item pick lists.

robots to minimize the time required to retrieve a collection of items from within a warehouse. An MILP formulation of this problem was presented and was utilized to examine the interactions between warehouse configurations and the composition of the fleet of order-picking robots.

The numerical analysis provided several key insights. From a warehouse layout perspective, robot order-pickers offer the greatest improvements over traditional human-picker operations when the ratio of ADBPL to ADFD is higher. This occurs when there are more PAs or fewer CAs. Furthermore, centrally-located packing stations lead to consistently better performance over packing stations located on the periphery of the warehouse.

From the perspective of configuring the robot fleet, in general, increasing the picker robot's grasp-and-place speed is more beneficial when the ADFD and ADBPL are both low or there are sufficient numbers of pickers and high-capacity transporters. When there are numerous low-capacity transporters but relatively few pickers, adding a picker is preferable if the ADBPL is high; increasing the grasp-and-place speed of a picker robot is preferable if ADBPL is low. If there are numerous pickers but few low-capacity transporters, adding transporter capacity is more effective when ADFD is high; adding a transporter is more effective when ADFD is low. When the numbers of pickers and low-capacity transporters are both sufficient, adding transporter capacity is preferable if the ADFD is high, but increasing grasp-and-place speed is preferable if the ADFD is low. Additionally, the impacts of the item grasp-and-place speeds decrease as the number of pickers increases or as the size of the warehouse increases, and the impact of the transporters' capacities decreases as the number of transporters increases. Furthermore, the impact of transporter capacities increases for larger warehouses.

This work provides a foundation for a variety of future research opportunities. For example, an extension to the PPT-VRP that considers aisle congestion could be of importance to practitioners. While the proposed model is appropriate for analyzing the relationships between layout configurations and robot system parameters, a separate approach is required to monitor and guide the robots in action. This might take the form of dispatching rules that can be implemented dynamically, taking into consideration the variability in the robots' pick and place times. Due to the NP-hard nature of the PPT-VRP, large-scale pick lists were not considered in the analysis of optimal solutions in this study. Therefore, efficient heuristics for large-scale problems are desirable. From a warehouse operations perspective, an analysis of different storage policies, to extend beyond the uniform policies considered herein, would be valuable. The determination of suitable order-batching methodologies remains an open topic in the context of robot-based warehousing. Another related problem is the use of robots for re-stocking activities, where transporter robots bring items from the depot back into the warehouse and picker robots return items to the shelves.

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